

COMMENTS ON “TOPICS IN
ABSOLUTE ANABELIAN GEOMETRY II:
DECOMPOSITION GROUPS AND ENDOMORPHISMS”

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(1.) In the situation of Example 3.2, (ii) [cf. also Corollaries 3.3, 3.4], we observe that it follows immediately from the *k*-*coricity* condition in Definition 3.1, (a), that the *open subscheme* [i.e., strictly speaking, *open substack*] $U_X \subseteq X$ is *completely determined* by the positive integer N and, in particular, *independent* of the choice of finite étale coverings $V \rightarrow X$, $V \rightarrow D$. In fact, this independence is *implicit* in the statement of Corollary 3.4.

(2.) In the statement of Corollary 3.4, the phrase “**pro-finite étale covering of X** ” should read “**pro-finite étale covering of X_i** ”.

(3.) In the Π -*chain* discussed in Corollary 3.7, (a), it is to be understood that the profinite group “ Π_j ” [cf. the notational conventions of [AbsTopI], Definition 4.2, (iii)] that corresponds to the *input* datum for the final “ \wedge ” in the associated type-chain — i.e., the profinite group that corresponds to the copy of the projective line minus three points “ P ” of Example 3.6, (ii) — is such that

- the corresponding “geometric fundamental group Δ_j ” [cf. the notational conventions of [AbsTopI], Definition 4.2, (iii)] is a *profinite free group of rank 2*;
- the set of *conjugacy classes of cuspidal decomposition groups* in Δ_j is of *cardinality 3*

— i.e., such that Π_j does indeed correspond to a copy of the projective line minus three points [cf. [AbsTopI], Lemma 4.1, (iv); [AbsTopI], Lemma 4.5, (v)]. Note that, by [AbsTopI], Lemma 4.5, (v), these conditions on Π_j are *entirely group-theoretic*. Although these conditions were not stated explicitly in Corollary 3.7, (a), they were intended to be *implicit* in the phrase “*which admits an **entirely ‘group-theoretic’ description***”, together with the reference given in Corollary 3.7, (a), to Example 3.6, (ii).

(4.) In the final paragraph of the proof of Corollary 2.10, the text “*totally ramified at precisely one closed point but unramified elsewhere*” should read “*totally ramified at some closed point*”.

(5.) In the second to last sentence of the Introduction, the word “notatable” should read “notable”.

(6.) In item (b) of the proof of Proposition 1.3, (viii), the notation “ $C \stackrel{\text{def}}{=}$ ” should be replaced by “ $C =$ ”.

(7.) In the second sentence of the proof of Proposition 2.5, (vi), the notation “ V_x ” should be replaced by “ \underline{V}_x ”.

(8.) In the fifth sentence of the proof of Proposition 2.5, (xi), the notation “ $W^{\log} \rightarrow V^{\log}$ ” should be replaced by “ $W_{\bullet}^{\log} \rightarrow V_{\bullet}^{\log}$ ”.

(9.) In the proof of Lemma 2.6, the phrase “*totally wildly ramified*” is to be understood as referring to a situation in which one considers a finite extension field of the ring of fractions of a discrete valuation ring \mathcal{O} such that the integral closure \mathcal{O}' of \mathcal{O} in the finite extension field is itself a discrete valuation ring whose residue field is a *purely inseparable* extension of the residue field of \mathcal{O} .

(10.) In items (c), (d) at the beginning of the proof of Lemma 2.6, (iv), the notation “ C ” should be replaced by “ C_W ”.

(11.) In the text immediately following the final display in the proof of Corollary 2.9, the phrase “in $H^1(G_{k_i}, T_i)$ with the image of $H^1(G_{k_i}, T'_i)$ of” should read “in $H^1(G_{k_i}, T_i)$, with the image of $H^1(G_{k_i}, T'_i)$, of”.

Bibliography

- [AbsTopI] S. Mochizuki, Topics in Absolute Anabelian Geometry I: Generalities, *J. Math. Sci. Univ. Tokyo* **19** (2012), pp. 139-242.